Introduction

The function of a crossover is to separate the input voltage to the different speakers or drivers (subwoofer, woofer and tweeter) depending on the frequency. Crossovers can be classified as active, meaning that they need to be powered and they filter the signal before the signal is amplified, and passive, they don't need to be powered and they filter the signal after this is amplified. The crossovers can also be classified by the order, first, second, third, fourth or higher.

I designed a 2^{nd} order 2-way (woofer and tweeter) passive crossover, I will derive the equations, implement the crossover, and analyze its response.

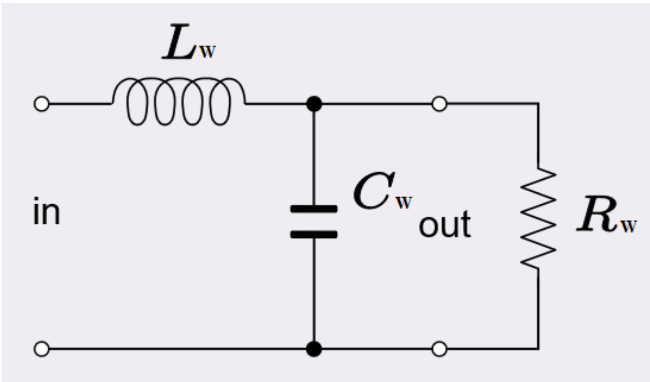
Before designing a crossover, the most important part is knowing the specs of your woofer and tweeter. Know the **impedance** of both drivers (R_W and R_T), and the frequency response of both drivers. The frequency response will help us pick the crossover frequency, this should be higher than the minimum frequency of the tweeter, and lower than the maximum frequency of the woofer.

Also, it is very important to pick two drivers with a **similar sensitivity** (+-5 dB difference), i.e. dB 1W/1m or dB 2.83V/1m. The sensitivity defines how a driver converts power (watts) into volume (decibels). It is worth explaining that with 8 ohm drivers you will find the sensitivity as dB 1W/1m, while with 4 ohm and 6 ohm drivers the sensitivity is written as dB 2.83V/1m. The explanation is the following, with an 8 ohm drivers, the voltage needed to generate one watt is 2.83V ($P = V^2/R$), hence, dB 1W/1m and dB 2.83V/1m are equivalent; however, if the driver has an impedance of 6 ohm the power generated with 2.83V is 1.33W.

In my case, I picked a woofer with an impedance of 8 ohm, and a sensitivity of 83.5 dB 1W/1m, and a tweeter with an impedance of 6 ohm, and a sensitivity of 88 dB 2.83V/1m. If the crossover has an input signal of a sine wave with low frequency, and an amplitude of 1V, the crossover will send the signal entirely to the woofer, the intensity going through the woofer will be 0.125A (V/R) and this will generate a sound of 29 dB per meter (83.5 db 2.83V/1m · 1V/2.83V); if the signal has a high frequency, it will be sent to the tweeter, the intensity will be higher, 0.167A, and the sound will be 31 dB per meter loud. Thus, we can conclude that for two drivers with similar sensitivities, if one of the drivers has a lower impedance, for the same amplitude of signal (voltage), the driver with lower impedance will need more intensity and more power to generate the same volume of sound. This also explains why in the specs of amplifiers, the maximum power is something like 30W for 8 ohm/ 60W for 4 ohm; this doesn't mean that the 4 ohm driver will sound louder, it means that the amplifier magnifies the voltage of the input signal by a maximum factor, independently if the drivers have an impedance of 8 or 4 ohms, and to amplify the voltage up the maximum factor for the 4 ohm driver, the amplifier will consume twice the amount of power.

Circuits

A second order low pass filter LCR circuit can be simplified the following way:

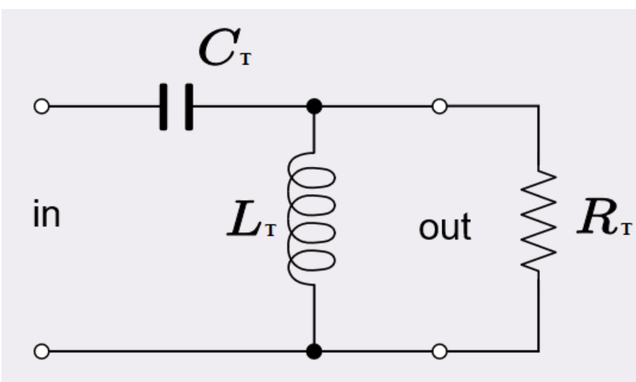


Where R_W represents the woofer's impedance.

The transfer function of this circuit is the following :

$$\frac{V_{out}}{V_{in}} = \frac{1}{C_W L_W S^2 + \left(\frac{L_W}{R_W}\right) S + 1} \tag{1}$$

And a high pass filter can be represented as:



Where R_T represents the tweeter‘s impedance.

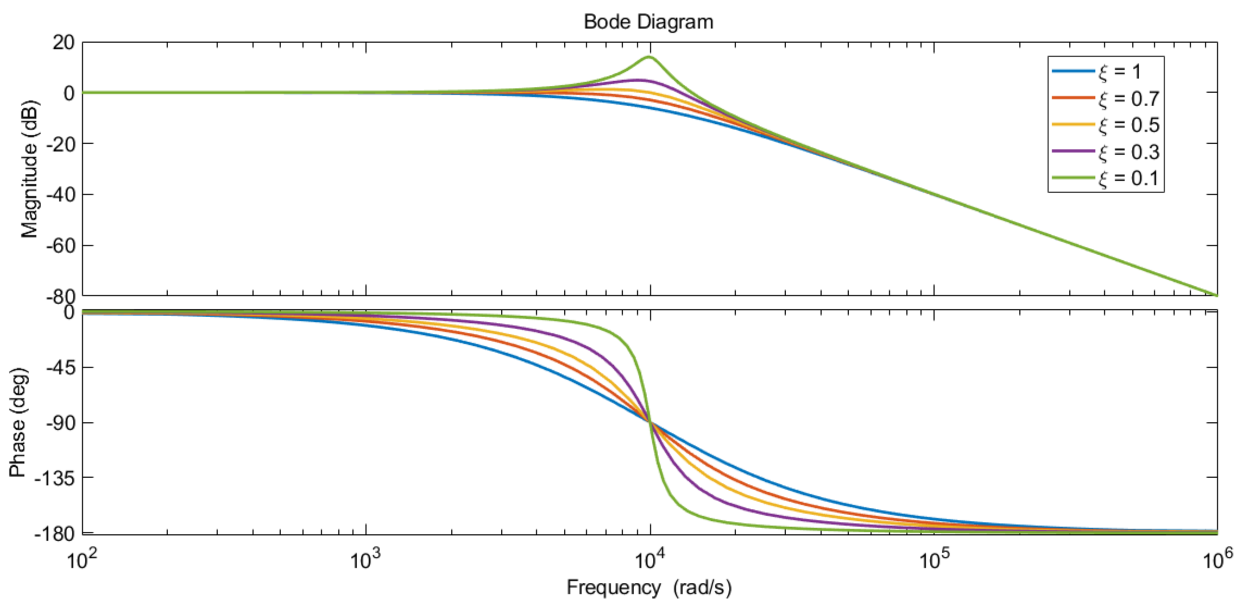
The transfer function of this circuit is the following :

$$\frac{V_{out}}{V_{in}} = \frac{C_T L_T S^2}{C_T L_T S^2 + \left(\frac{L_T}{R_T}\right) S + 1} \tag{2}$$

If we look at equations (1) and (2), we can identify that they are a 2[nd] order damped system, these system has the following transfer function:

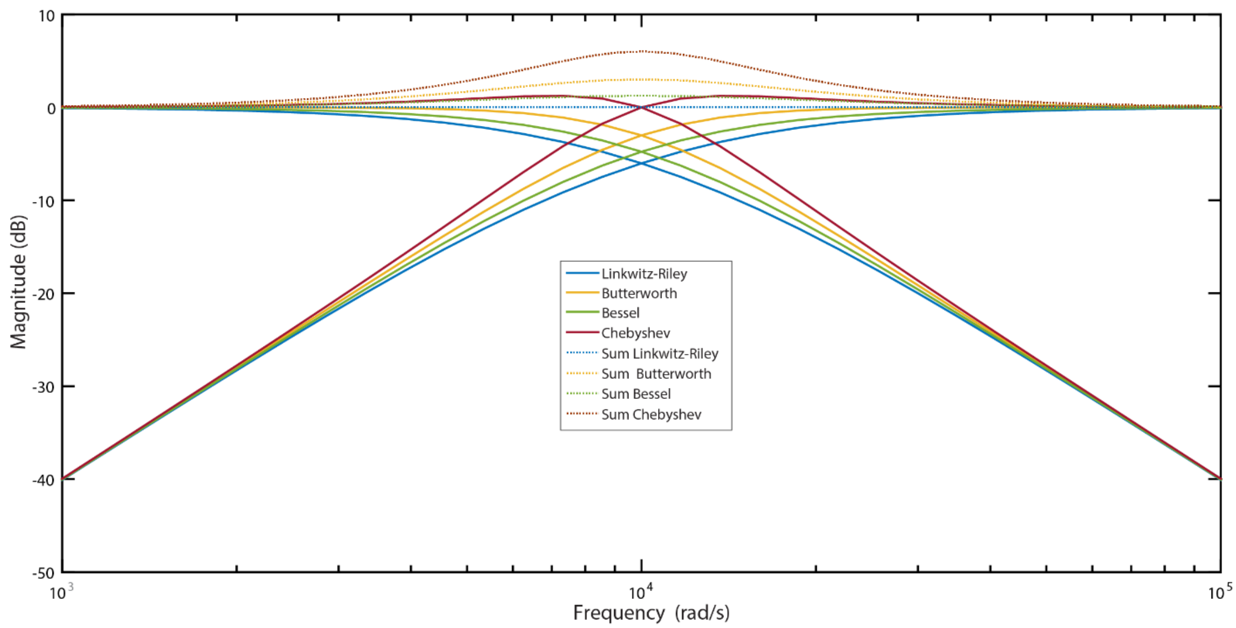
$$\frac{Y}{F} = \frac{1}{\frac{S^2}{\omega_n{}^2} + \frac{2\xi S}{\omega_n} + 1}$$
$$\frac{Y}{F} = \frac{S^2/\omega_n{}^2}{\frac{S^2}{\omega_n{}^2} + \frac{2\xi S}{\omega_n} + 1} \quad (3)$$

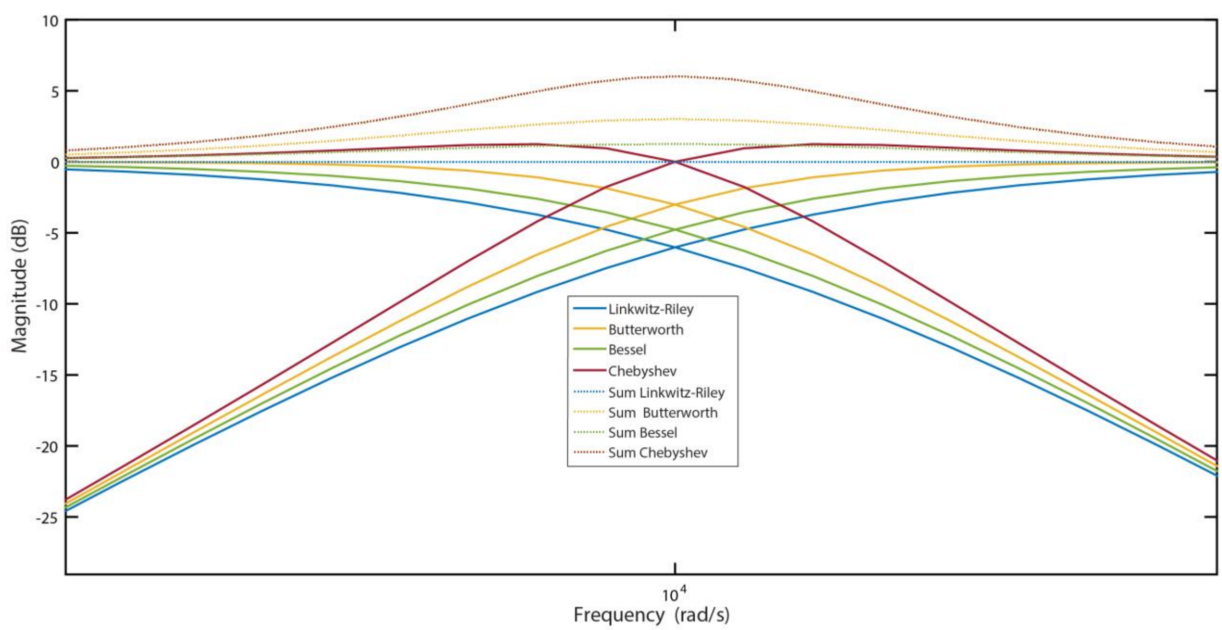
Where ω_n is the natural frequency, and ξ is the damping ratio. I have plotted the frequency response of a low pass filter, with a ω_n=10,000 rad/s and with different damping ratio.



It can be seen that the amplitude decays after the natural frequency, and that for damping ratios higher than 0.5 no amplitude peak is observed around the natural frequency. We can now explain and classify the different types of passive 2nd order filters depending on the damping ratio.

The 4 main types of filters are: Linkwitz-Riley, with a damping ratio of $\xi = 1$ and a Q factor of $Q = 1/(2\xi) = 0.5$; Butterworth, with a damping ratio of $\xi = 0.866$ and a Q factor of $Q = 0.577$; Bessel, with a damping ratio of $\xi = 0.707$ and a Q factor of $Q = 0.707$; and Chebyshev, with a damping ratio of $\xi = 0.5$ and a Q factor of $Q = 1$. The frequency response of the four filters, with a crossover frequency of ω_n=10.000 rad/s, are depicted in the following figure, along with the sum of the amplitude of the crossover, i.e. the sum of the amplitudes of the low pass filter and the high pass filter.





It can be seen that the Linkwitz-Riley has a flat response, i.e. the sum is constant, the Butterworth has a peak of 1.5 dB at the crossover frequency, the Bessel has a peak of 3 db, and the Chebyshev has a peak of 6 db. One could think that the Linkwitz-Riley is always the best option, since there are no peaks of amplitude on the crossover frequency; however, the response of the drivers might not be constant throughout the frequency spectrum, and the amplitude might decay or be lower around the crossover frequency, this is why the amplitude peaks of the different filters can compensate this problem. This is why, to determine the best crossover, it is important to look at the frequency response plots (decibels vs. frequency) that are available in the specifications of the drivers.

Equations

Using equations (1), (2) and (3), we can find the capacitance and inductance needed for our crossover. It is worth mentioning that we want both high pass and low pass to be symmetric, hence, they have to have the same natural frequency and the same damping ratio.

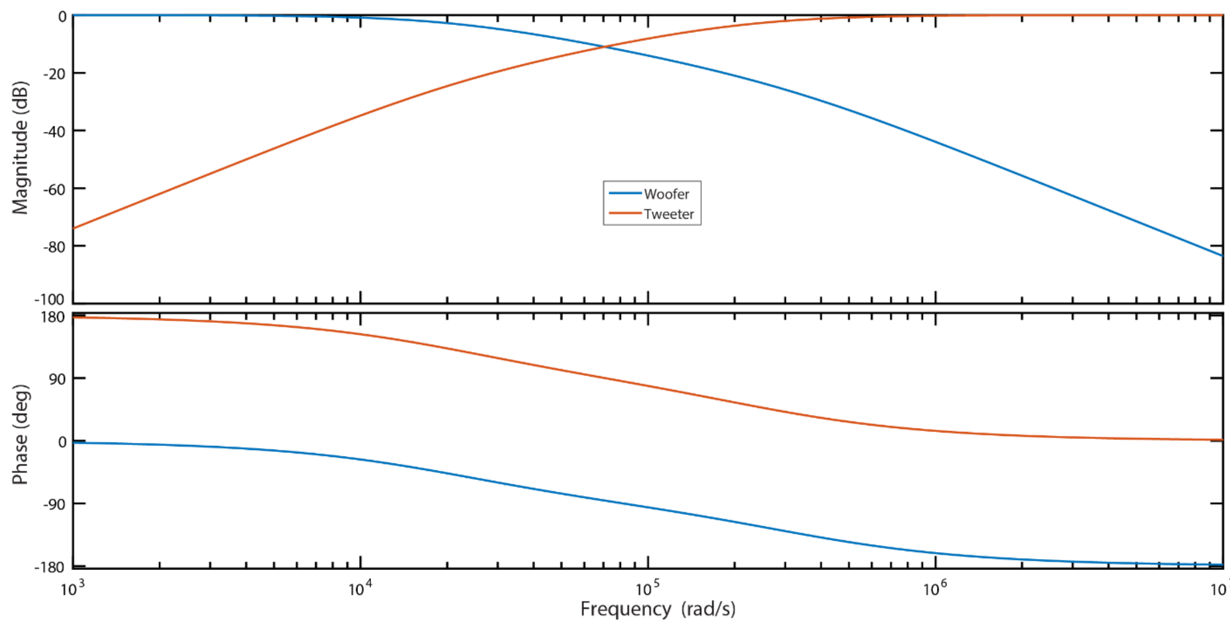
$$\begin{aligned} \omega_n &= 2\pi f_n = \frac{1}{\sqrt{L_W C_W}} = \frac{1}{\sqrt{L_T C_T}} \\ \xi &= 0.5\omega_n \frac{L_W}{R_W} = 0.5\omega_n \frac{L_T}{R_T} \\ L_W C_W &= L_T C_T \\ \frac{L_W}{R_W} &= \frac{L_T}{R_T} \end{aligned} \tag{4}$$

In my case I picked a frequency of 4600 Hz and a damping ratio of 0.7, Bessel filter. I get:

$$\begin{aligned} L_W &= \frac{2\xi R_W}{\omega_n} = \frac{2 \cdot 0.7 \cdot 8}{2\pi \cdot 4600} = 0.388 mH \\ L_T &= R_T \frac{L_W}{R_W} = 6 \frac{0.388 mH}{8} = 0.291 mH \\ C_W &= \frac{1}{\omega_n^2 L_W} = \frac{1}{\left(2\pi \cdot 4600\right)^2 \cdot 0.388 \cdot 10^{-3}} = 3.08 \mu F \\ C_T &= \frac{L_W C_W}{L_T} = \frac{0.388 \cdot 10^{-3} \cdot 3.08 \cdot 10^{-3}}{0.291 \cdot 10^{-3}} = 4.11 \mu F \end{aligned} \tag{5}$$

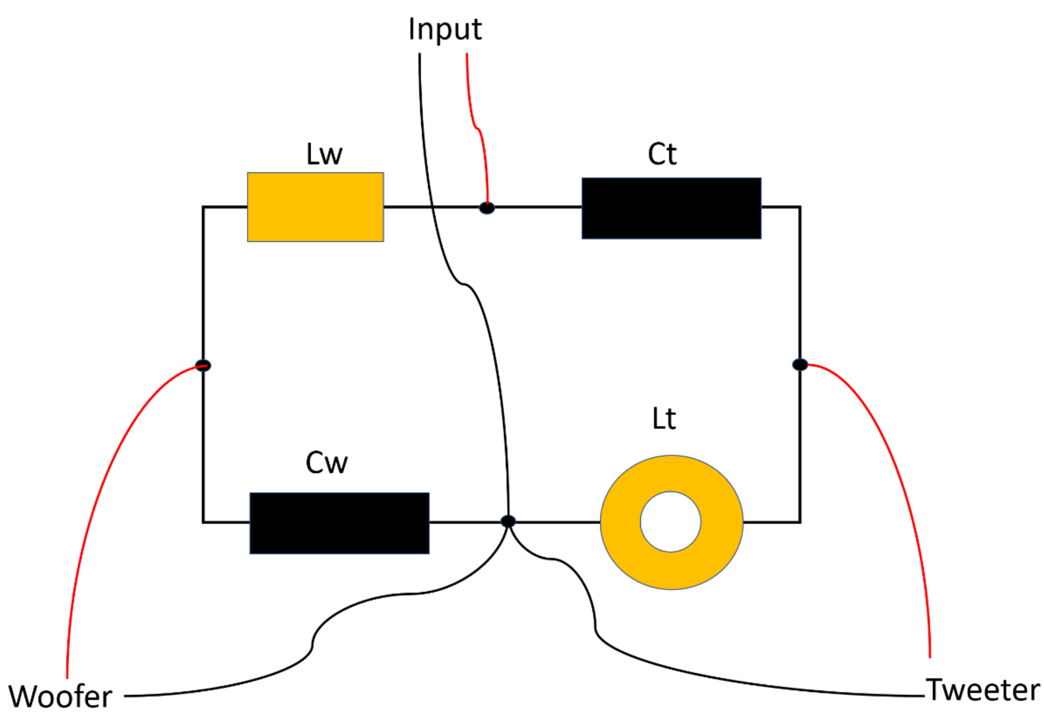
I can round it up to L_W = 0.4 mH, L_T = 6·0.4/8 = 0.3 mH, C_W = 3 μF, and C_T = 0.4·3/0.3 = 4 μF. Then fn = 4594 Hz, and ξ = 0.72.

The frequency response of the crossover is depicted in the following plot:

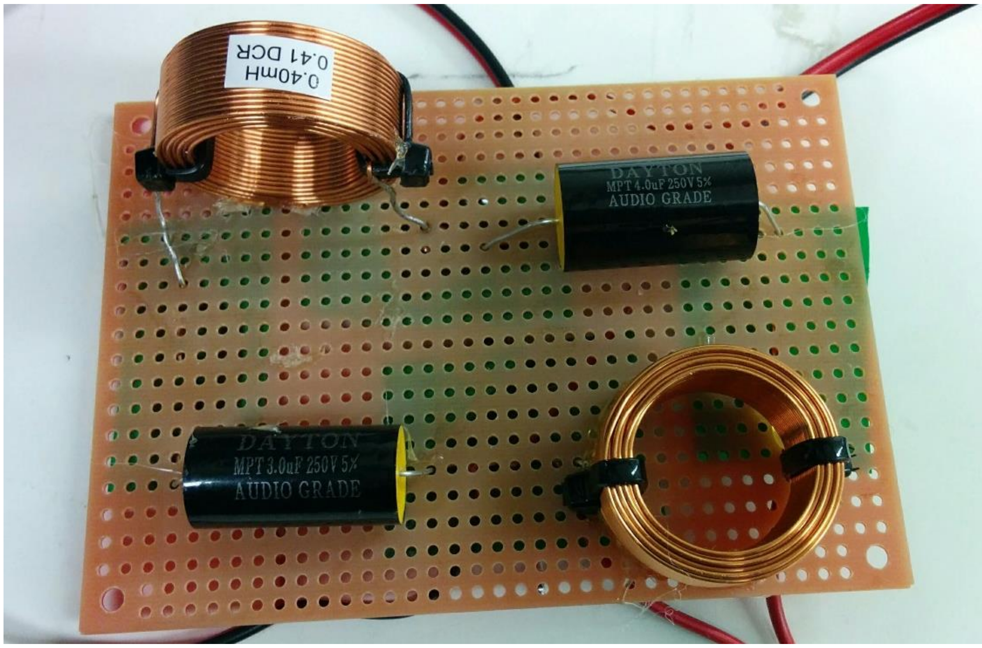


Implementation and testing

I proceeded to assembly and solder the crossover. The schematics of the wiring is depicted in the following image.

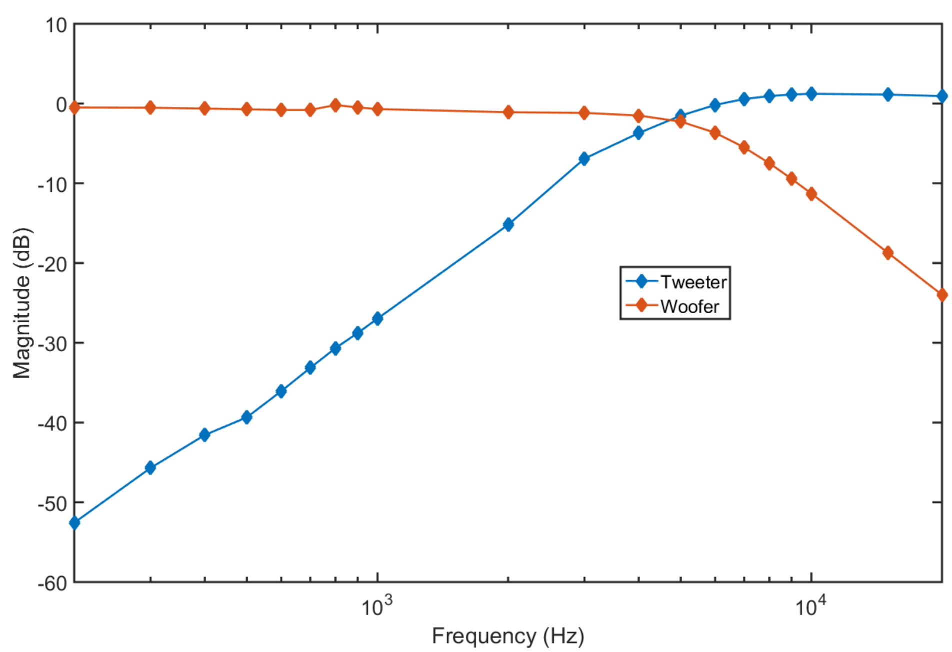


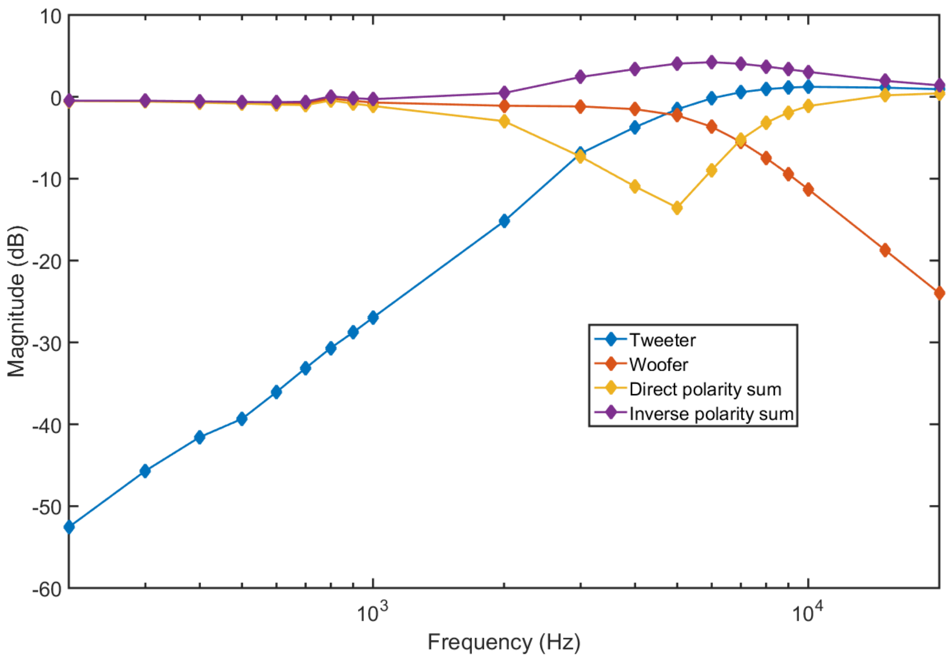
The following image is a picture of the final crossover.



To avoid interference between the magnetic fields, the coils should be placed as separate as possible; and through the hole of one coil you shouldn't see the other one, in other words, the axes of both coils should not cross. For more information about the placement of the coils, check http://www.troelsgravesen.dk/coils.htm

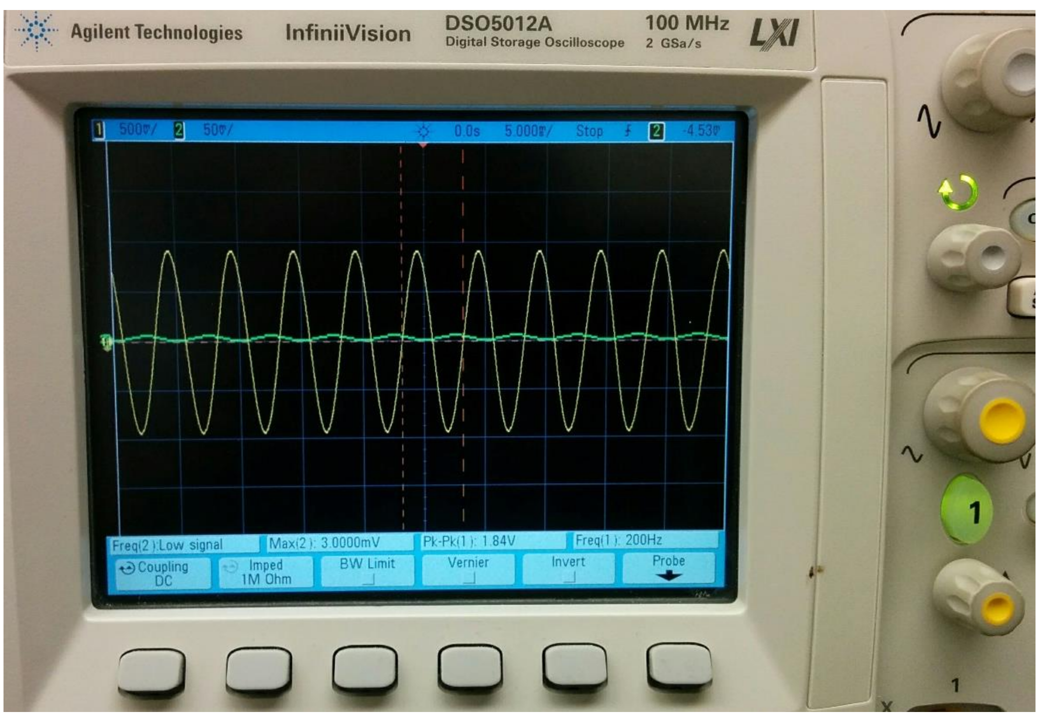
I tested the crossover using a function generator and an oscilloscope. I swept different frequencies of input signals, and measured the amplitude (voltage) for both the output of the woofer (low pass filter) and the output of the tweeter (high pass filter). The results can be seen in the following plot:



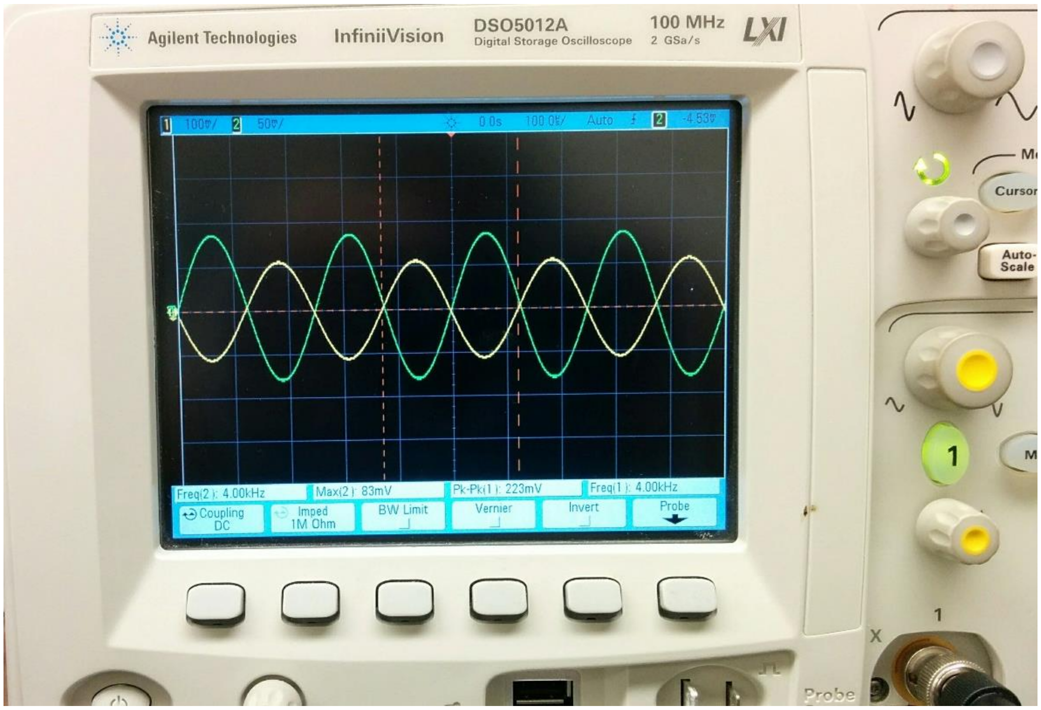


I will invert the polarity of the tweeter in order to have the desired performance.

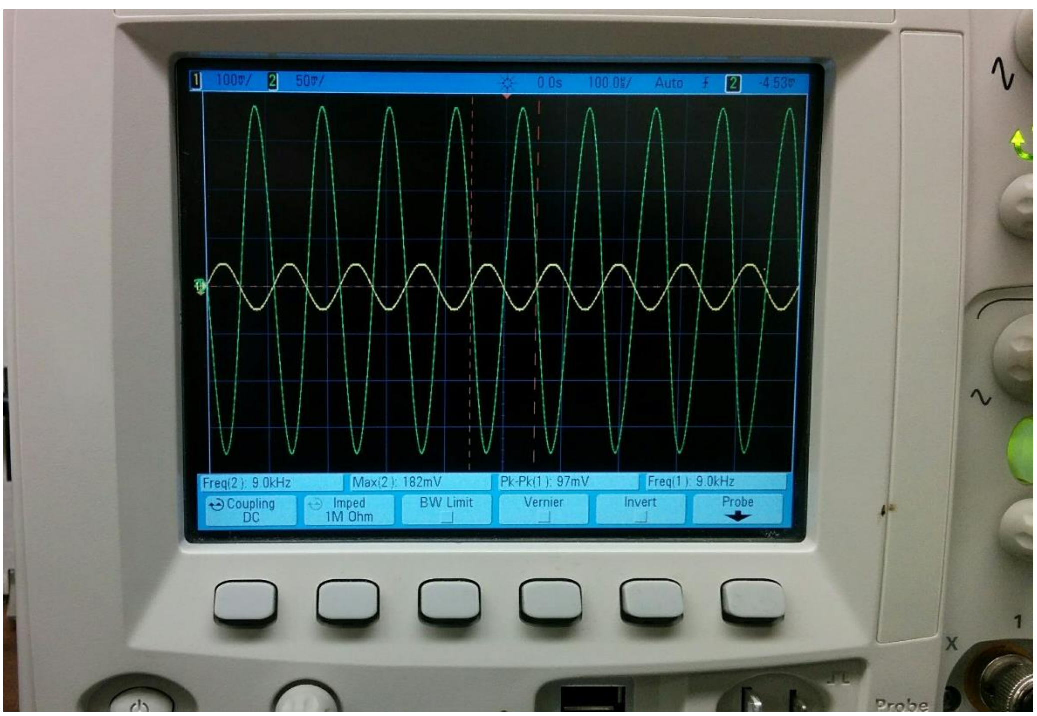
And here are some pictures of the oscilloscope during the test, in yellow the output of the woofer is represented, and in green the output of the tweeter is represented:



Frequency of 200 Hz



Frequency of 4,000 Hz



Frequency of 9,000 Hz